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ON CRYOGENIC LIQUID POOL EVAPORATION

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Summary

In this paper we evaluate the rate of evaporation from two typical configurations of cryogenic liquid spills on a solid homogeneous surface. Only theoretical results are given, and the results are valid only when the liquid is boiling as a result of heat transfer from the underlying ground surface.

1. Introduction

Various industrial gases are stored in liquified form, either under high pressure and at ambient temperature or under less pressure and at lower temperatures. The first type of storage gives rise to quite spectacular releases during loss of containment, involving formation of large quantities of aerosol. Release in the latter type is more gentle, the main result being release of cold vapours (at boiling point, 1 atm) as a result of contact with the surface, which will normally be much warmer than the cryogenic liquid. The second type of storage is the subject of this paper.

In the later stage of this type of release, the transfer of sensible heat from the air becomes important as the heat transfer from the soil becomes sufficiently low. This problem requires consideration of the structure of the turbulent air flow above the pool and is not considered in this paper. We further limit ourselves to cases in which the liquid is boiling, as this involves a simple relationship between the amount of vapour production and the magnitude of the heat flow from the ground. Thus,

$$q = \frac{\lambda}{L}\frac{\partial T}{\partial z}, \qquad (1)$$

where q is the rate of evaporation, L is the latent heat, λ is the thermal conductivity of the homogeneous ground and $\partial T/\partial z$ is the vertical temperature gradient at the surface of the ground. In eqn. (1) we assume that the pool is large and "edge effects" can therefore be neglected.

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The magnitude of $\partial T/\partial z$ can be found by solving the heat-conduction equation using the boundary condition that the surface temperature instantaneously decreases by the amount ΔT, the temperature difference between the ground and the cryogenic liquid. As we have restricted ourselves to cases where the liquid boils, ΔT is a constant. The solution to this classic problem, sometimes called Stokes' 1st problem, is

$$\frac{\partial T}{\partial z} = \Delta T \sqrt{\frac{\rho c}{\lambda \pi t}} , \tag{2}$$

where ρ and c are the density and specific heat capacity of the soil, respectively, and t is the time since the temperature was changed. Combination of eqns. (1) and (2) gives

$$q = \frac{s}{\sqrt{t}} , \tag{3}$$

where $s = (\Delta T/L)\sqrt{\rho c \lambda/\pi}$ contains all the physical parameters in the problem. According to eqn. (3), the rate of evaporation decreases with the square root of time. Typical values ($\rho = 2 \times 10^3$ kg m^{-3}, $c = 10^3$ J kg^{-1} K^{-1}, $\lambda = 1$ W m^{-1} K^{-1}, $\Delta T = 50$ K, and $L = 1000$ kJ kg^{-1}) give an evaporation rate of 40 g m^{-2} divided by the square root of t in seconds. Equation (3) is in quite good agreement with experimental results [1], although additional effects can occur with spills on natural surfaces, e.g., percolation into loose material and ice formation in soil water.

However, the main problem in using eqn. (3) in practice is that it initially gives an infinitely large evaporation rate. In the past this has been remedied in various ways, such as, for example, prescribing an initial flash-off of a fixed part of the released material and then using eqn. (3) only after an arbitrary time lapse, typically of a few seconds.

In this paper we propose a more rational procedure based on a consideration of the finite time in which the liquid spreads from its source to the final pool size.

2. The initial evaporation phase

To estimate realistically the rate of evaporation from a cryogenic spill, we must realise that the area of the pool increases as a function of time, and that the area is zero at zero time.

To get the average evaporation rate for the entire pool at time t, it is necessary (as an approximation) to consider q for each differential area element $\mathrm{d}A$ which became covered with fluid during time $t' + \mathrm{d}t$, and then sum to the desired time t. In other words, we must account for the "boiling time" of each elemental area. Thus, for the first differential time period

$$Q_1 = \mathrm{d}A_1 \frac{s}{\sqrt{\mathrm{d}t}} , \tag{4}$$

where dA_1 is the ground area which became covered by the pool during time dt. In the next differential period the evaporation becomes

$$Q_2 = dA_1 \frac{s}{\sqrt{2dt}} + dA_2 \frac{s}{\sqrt{dt}} \,, \tag{5}$$

where the rate of evaporation from the first element now has decreased by a factor of $\sqrt{2}$. In general,

$$Q_n = dA_1 \frac{s}{\sqrt{ndt}} + dA_2 \frac{s}{\sqrt{(n-1)dt}} + \ldots + dA_n \frac{s}{\sqrt{dt}} \,, \tag{6}$$

which gives the evaporation rate from the pool at time $t = n\,dt$.

The use of eqn. (2), which is a solution to a one-dimensional problem, in connection with the differential elements in eqn. (4) needs to be justified. Although horizontal temperature gradients are present in the ground, they are small compared to $\partial T/\partial z$, except for a narrow zone of width x close to the spreading pool front. The extent of this zone can be estimated from $\partial T/\partial x \sim \Delta T/x \ll \partial T/\partial z$. Using eqn. (2), this condition is equivalent to

$$\sqrt{\frac{vx}{D}} \gg 1, \tag{7}$$

where t has been replaced by x/v, v being the velocity of the advancing pool front, and the diffusivity $D = \lambda/\rho c$. With D typically 10^{-6} m^2 s^{-1} (see values given above) and a very low estimate of v of $\sim$0.1 m s^{-1} (more typically 1 m s^{-1} or larger), the maximum width in which $\partial T/\partial x$ is not much less than $\partial T/\partial z$ is $\sim$1 cm, an insignificant value compared to pool diameters of meters.

Before the sum in eqn. (6) can be evaluated, we must assume a model for the area of the pool as a function of time. As an example, we choose the case of a continuous point source under calm conditions. The radius of the ensuing pool is given by [2]

$$r = \left(\frac{g\dot{V}}{2\pi}\right)^{1/4} t^{3/4} \,, \tag{8}$$

where g is acceleration due to gravity and $\dot{V}$ is the release rate of fluid. The size of the elemental area is $dA_n = 2\pi r\, dr$ and, using r given by eqn. (8) at time $t = n\,dt$, dA_n becomes

$$dA_n = \tfrac{3}{4} (2\pi g\dot{V})^{1/2} \sqrt{ndt}\, dt. \tag{9}$$

Insertion of eqn. (9) into eqn. (6) gives

$$Q_n = sC\left[\frac{\sqrt{1}}{\sqrt{n}} + \frac{\sqrt{2}}{\sqrt{n-1}} + \frac{\sqrt{3}}{\sqrt{n-2}} + \ldots + \frac{\sqrt{n}}{\sqrt{1}}\right]\frac{t}{n} \,, \tag{10}$$

where we, for convenience, have introduced the constant $C = \tfrac{3}{4} (2\pi g\dot{V})^{1/2}$.

The sum S_n in eqn. (10) may be estimated by integration. Thus

$$S_n \cong \int_1^n \frac{\sqrt{x}}{\sqrt{n+1-x}}\,dx, \tag{11}$$

resulting in

$$S_n = \left[\tan^{-1}\sqrt{n} - \tan^{-1}\frac{1}{\sqrt{n}}\right](n+1). \tag{12}$$

For large n ($n \gg 1$) S_n equals $(\pi/2 - 0)n$, so that eqn. (10) becomes

$$Q_n = sC\frac{\pi}{2}t. \tag{13}$$

This shows the interesting effect that, for the case of a continuous spill, the evaporation rate does not begin as infinitely large, but, in fact, is proportional to the time.

A result analogous to eqn. (13) can be obtained from a simple scale analsis. From eqns. (3) and (8)

$$\left.\begin{aligned} q &\sim t^{-1/2} \\ r &\sim t^{3/4} \end{aligned}\right\}, \tag{14}$$

which gives $Q(t) \sim \pi r^2 q \sim t^{3/2} t^{-1/2} = t$. In such a consideration the proper value of the proportionality between Q and t cannot be given. Thus, in the latter crude analysis, the factor $sC\pi/2$ is underestimated by a factor $4\sqrt{2}/3\pi \cong 0.6$.

3. The maximum evaporation rate

If the expansion of the pool is limited by a dyke (circular, concentric), the time development as given by eqn. (13) stops when at time t_d the diameter of the pool becomes equal to the dyke diameter. After this time the evaporation rate begins to decline as a consequence of the progressive cooling of the substrate, a development which will approximately follow eqn. (3). The evaporation rate for this release case is shown as a function of time in Fig. 1. The maximum evaporation rate is calculated from eqn. (13), with t computed from eqn. (8), in which r is the radius of the circular dyke.

If the spreading of the pool is not limited by a dyke, we can make the following qualitative consideration: when the rate of evaporation, which is increasing linearly with time, has become so large that $Q \sim \rho_L \dot{V}$, i.e., that the rate of evaporation equals the source strength, the size of the pool itself is limited. This leads to

$$r = \frac{\rho_L^{3/4}\,\dot{V}^{5/8}}{g^{1/8}\,s^{3/4}} \tag{15}$$

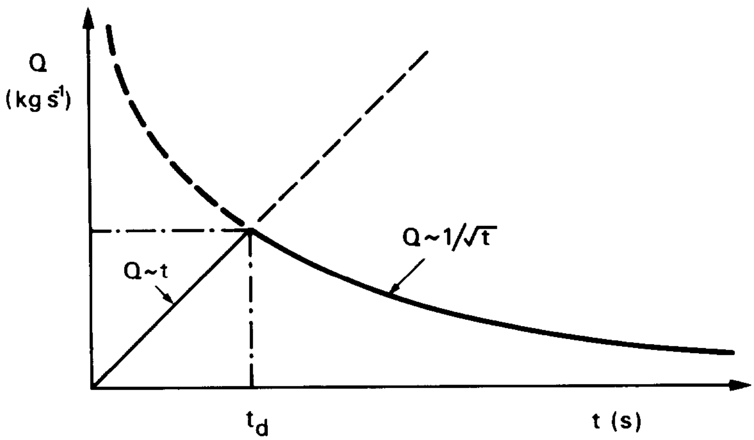

Fig. 1. The rate of evaporation as a function of time for a pool resulting from a continuous release of cryogenic liquid. The point t_d corresponds to the time when the pool has reached its maximum area.

as an estimate of the maximum extent of an unconfined pool resulting from a continuous release. The corresponding estimate of the maximum evaporation rate is obtained from eqn. (13) with $t = t_d$ calculated from eqns. (8) and (15).

Another typical release configuration in accident scenarios is the instantaneous release of a certain volume V. The gravity spreading in this case is such that [3]

$$r \cong (gV)^{1/4} t^{1/2}. \tag{16}$$

Bypassing the mathematics of section 2 and going directly to a scale analysis leads to an evaporation rate $Q(t) \sim \pi r^2 q \sim t\, t^{-1/2} = t^{1/2}$, or with the physical constants included,

$$Q(t) \cong s\sqrt{gV}\,\pi t^{1/2}. \tag{17}$$

Compared to eqn. (13), this expression gives a somewhat slower increase with time of the evaporation rate. Again the presence of a boundary or dyke leads to a straightforward estimate of the maximum evaporation rate. If the spreading is not limited, a qualitative argument leading to a limit can be posed as

$$Qt_d \sim \rho_L V, \tag{18}$$

which simply equals the total released mass with an estimate of the total evaporation up to time t_d. This leads to the following estimate of t_d:

$$t_d = \frac{\rho_L^{2/3} V^{1/3}}{g^{1/3} s^{2/3}}, \tag{19}$$

with a corresponding maximum radius of spread equal to

$$r = \frac{\rho_L^{1/3} V^{5/12} g^{1/12}}{s^{1/3}}. \tag{20}$$

An estimate of the corresponding maximum evaporation rate is again readily obtained using eqn. (17) with $t = t_d$ from eqn. (19).

Expressions similar to eqns. (19) and (20) can be obtained from reworking eqns. (28) and (29) of Ref. [4] (note the difference in notation), which were derived in an entirely different manner.

4. Conclusions

In this paper we demonstrate that the expected variation with time of the rate of evaporation from spills (continuous or instantaneous) of cryogenic liquids is a smooth function which starts from zero at $t = 0$, reaches a maximum at a characteristic time t_d, and then follows the classical $t^{-1/2}$ relationship.

For pools spreading unbounded, there are differences between a continuous and an instantaneous release regarding the dependence of the maximum radius on the various physical parameters of the problem. A comparison of eqns. (15) and (20) shows that r for an instantaneous release is much less dependent on s, (that is, on the thermal properties of the substrate and the temperature and latent heat of vaporization of the liquid) than for a continuous release. The dependence on the liquid density is also less for an instantaneous release.

List of symbols

A	area of pool [m^2]
c	specific heat capacity of the ground [$J\ kg^{-1}\ K^{-1}$]
C	$\equiv \frac{3}{4}(2\pi g\dot{V})^{1/2}$ [$m^2\ s^{-3/2}$]
L	latent heat of evaporation [$J\ kg^{-1}$]
q	rate of evaporation [$kg\ s^{-1}\ m^{-2}$]
Q	rate of evaporation from entire pool [$kg\ s^{-1}$]
r	radius of pool [m]
s	$\equiv (\Delta T/L)\sqrt{\rho c \lambda/\pi}$ [$kg\ m^{-2}\ s^{-1/2}$]
t	time [s]
t_d	time-scale for pool development [s]
T	soil temperature [K]
ΔT	initial difference between ground temperature and temperature of cryogenic liquid [K]
V	instantaneous release volume [m^3]
$\dot{V}$	continuous release rate of fluid [$m^3\ s^{-1}$]

λ thermal conductivity of the ground [W m^{-1} K^{-1}]
ρ density of ground material [kg m^{-3}]
ρ_L density of cryogenic liquid [kg m^{-3}]

Other symbols are defined as they are encountered in the text.

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